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GUT with Anomalous $U(1)_A$ Suggests Heterotic M-Theory?

Nobuhiro MAEKAWA^a*Department of Physics, Kyoto University, Kyoto 606-8502, Japan*

Abstract

We show that a new GUT scenario we proposed indicates that heterotic M-theory is one of the most interesting possibility to describe our world because of the two features in the scenario. The first feature is that E_6 unified group plays an essential role not only in understanding larger mixing angles in lepton sector than in quark sector but also in solving the SUSY flavor problem by introducing non-abelian horizontal symmetry $SU(2)_H$ or $SU(3)_H$. The second feature is that the cutoff scale must be taken as the usual GUT scale $\Lambda_G \sim 2 \times 10^{16}$ GeV to realize natural gauge coupling unification. Because this cutoff scale is smaller than the Planck scale or string scale, it may suggest the existence of extra dimension in which only gravity modes can propagate. This talk is based on a dozen papers[1, 2, 3, 4, 5, 6, 7, 8, 9] some of which are collaborated with T. Yamashita, M. Bando and Q. Shafi.

^aE-mail: maekawa@gauge.scphys.kyoto-u.ac.jp

Table 1: Quantum numbers under $E_6 \times SU(2)_H \times U(1)_A \times Z_2$ are presented only for all non-singlet fields under $E_6 \times SU(2)_H$ except an doublet field $X(1, 2)_x$ which is required for Witten anomaly cancellation.

MATTER	$\Psi(\mathbf{27}, \mathbf{2})_4^+, \Psi(\mathbf{27}, \mathbf{1})_{3/2}^+$
HIGGS($SU(2)_H$)	$F(\mathbf{1}, \mathbf{2})_{-3/2}^+, \bar{F}(\mathbf{1}, \mathbf{2})_{-5/2}^+$
HIGGS $E_6 \rightarrow$ $SU(3)_C \times U(1)_{EM}$	$A(\mathbf{78}, \mathbf{1})_1^+, A'(\mathbf{78}, \mathbf{1})_5^+$ $\Phi(\mathbf{27}, \mathbf{1})_{-3}^+, C'(\mathbf{27}, \mathbf{1})_6^-$ $\bar{\Phi}'(\mathbf{27}, \mathbf{1})_5^+, \bar{C}(\mathbf{27}, \mathbf{1})_0^-$

1 Introduction and Overview of our scenario

One of the biggest difference between our GUT scenario and various previous GUT models is to introduce generic interactions (including higher dimensional interactions) with $O(1)$ coefficients in our scenario. Therefore, once the symmetry is fixed, the theory can be defined except the $O(1)$ coefficients. It is also quite amazing that almost all the problems can be solved with generic interactions; realistic quark and lepton masses and mixings (predicting LMA solution for the solar neutrino problem) can be obtained[1, 2]; doublet-triplet splitting can be realized without too rapid proton decay via dimension five operators[1, 3, 4]; this scenario gives a new natural explanation for the success of gauge coupling unification in the minimal SUSY standard model(MSSM) although E_6 gauge group, whose rank is larger than that of $SU(5)$, is adopted as the unified gauge group[5]; if SUSY breaking sector is introduced, μ problem is also solved[6]; if non-abelian horizontal gauge symmetry $SU(2)_H$ or $SU(3)_H$ is introduced, SUSY flavor problem is also solved.[7]

It is obvious that introducing generic interactions simplifies the process to obtain a realistic model from the superstring theory, because a symmetry defines a model. Let me show an example. Just the symmetry is presented in Table 1. Note that just two fields Ψ and Ψ_3 include all the three generation matter fields. Among the eight Higgs fields, only non-primed fields have non-vanishing vacuum expectation values (VEVs). And doublet Higgs in MSSM are included in Φ in this model. If the matter and Higgs ($SU(2)_H$) sectors change as in Table 2, all the three generation matter fields can be unified into a single multiplet $\Psi(\mathbf{27}, \mathbf{3})$ under $E_6 \times SU(3)_H$.

It is non-trivial that in these models, with generic interactions, doublet-triplet splitting is realized.[3, 4] Here we would like to concentrate on several points which suggest that the heterotic M-theory may be promising. In the scenario, E_6 (or E_8) GUT gauge group plays an important role in understanding larger neutrino mixing angles[2] and in solving SUSY flavor problem by introducing non-abelian horizontal gauge symmetry $SU(2)_H$ or $SU(3)_H$. [7] It is interesting

Table 2: Quantum numbers under $E_6 \times SU(3)_H \times U(1)_A \times Z_2$ are presented for matter and Higgs for $SU(3)_H$. The Higgs sector for breaking E_6 is the same as in Table 1. Cancellation of $SU(3)_H$ gauge anomaly requires some other fields, for example a **10** of $SU(3)_H$.

MATTER	$\Psi(\mathbf{27}, \mathbf{3})_{7/2}^+$
HIGGS($SU(3)_H$)	$F_i(\mathbf{1}, \mathbf{3})_{(-3,2)}^+, \bar{F}_i(\mathbf{1}, \bar{\mathbf{3}})_{(-2,-2)}^+ (i = 2, 3)$

that a structure in E_6 models simultaneously realizes two features, large neutrino mixing angles and suppressed flavor changing neutral currents (FCNC). Because $E_6 \times SU(3)_H$ is a maximal subgroup of E_8 , the heterotic string must be promising. Moreover, the scenario of GUT with anomalous $U(1)_A$ gauge symmetry generally explain why the three gauge couplings meet at a scale in MSSM.[5] This novel mechanism requires the cutoff scale around the usual GUT scale $\Lambda_G \sim 2 \times 10^{16}$ GeV, which are smaller than the Planck scale or the string scale. This may imply the extra dimension in which only gravity modes can propagate as discussed by Horava-Witten.[10]

2 Anomalous $U(1)_A$ gauge symmetry

Anomalous $U(1)_A$ gauge symmetry is a $U(1)$ gauge symmetry with gauge anomaly which is cancelled by Green-Schwarz mechanism.[11] But in our scenario, we use anomalous $U(1)_A$ gauge symmetry as just $U(1)$ gauge symmetry with Fayet-Illiopoulos D -term whose parameter ξ is smaller than the cutoff scale Λ . If a field Θ with the negative charge $\theta = -1$ is introduced, D -flatness condition for $U(1)_A$ $D_A = \xi^2 - |\Theta|^2 = 0$ determines the VEV as $\langle \Theta \rangle = \xi \equiv \lambda \Lambda$. (In this paper, we take $\lambda \sim \sin \theta_C \sim 0.22$.) Therefore, the anomalous $U(1)_A$ gauge symmetry is broken just below the cutoff scale.

Ibanez-Ross[12] pointed out that this field Θ can play the same role as the Froggatt-Nielsen field[13] in SUSY models. Assigning the anomalous $U(1)_A$ charges for quark, lepton and Higgs superfields, gauge invariant interactions become Yukawa interactions after developing the VEV $\langle \Theta \rangle \sim \lambda \Lambda$ as

$$W = \left(\frac{\Theta}{\Lambda} \right)^{q_i + u_j + h_u} Q_i U_j H_u \rightarrow \lambda^{q_i + u_j + h_u} Q_i U_j H_u, (i, j = 1, 2, 3), \quad (2.1)$$

where lowercase letters denotes the anomalous $U(1)_A$ charge and we use a unit in which $\Lambda = 1$. It is obvious that larger charge leads to smaller coupling and therefore smaller mass. And if we adopt $h_u = h_d = 0$, $(q_i) = (3, 2, 0)$, $(u_i) = (4, 2, 0)$, $(d_i) = (3, 2, 2)$, realistic quark masses and mixings can be obtained. Possible criticism for this obserbation may be that any mass spectrum (6 masses and 3 mixing)

can be obtained by assigning their 9 charges. However, this scenario generally gives Cabbibo-Kobayashi-Maskawa matrix as $(V_{CKM})_{ij} \sim \lambda^{|q_i - q_j|}$, which leads to a non-trivial relation $V_{13} \sim V_{12}V_{23}$ which is consistent with the experimental values.

How about in lepton sector? Because neutrino masses and mixings are obtained by

$$(m_\nu)_{ij} \sim \lambda^{l_i + l_j + 2h_u} \frac{\langle H_u \rangle^2}{\Lambda}, \quad (V_{MNS})_{ij} \sim \lambda^{|l_i - l_j|}, \quad (2.2)$$

there are more non-trivial relations in neutrino sector than in quark sector. One of them is

$$(V_{32})^2 \sim \frac{m_{\nu 2}^2}{m_{\nu 3}^2} \sim \frac{\Delta m_\odot^2}{\Delta m_{atm}^2} \equiv R. \quad (2.3)$$

The ratio R is dependent on the solutions for solar neutrino problem as

$$R \sim \lambda^2(\text{LMA}), \lambda^{3-4}(\text{SMA}), \lambda^6(\text{LMO}), \lambda^{11}(\text{VAC}). \quad (2.4)$$

It is obvious that the LMA solution for solar neutrino problem gives the best value of $V_{32} \sim 0.5$, which is near the experimental value $(V_{32}) \sim 0.7$. This has not been emphasized in the literature, though we pointed out that LMA solution is naturally realized in GUT with anomalous $U(1)_A$ symmetry[1, 2]

Because all the coefficients are determined by the anomalous $U(1)_A$ charges, the scales of VEVs are also determined by the charges (no flat direction) as

$$\langle O_i \rangle \sim \begin{cases} \lambda^{-o_i} & o_i \leq 0 \\ 0 & o_i > 0 \end{cases}, \quad (2.5)$$

where O_i are GUT gauge singlet operators (G -singlets) with anomalous $U(1)_A$ charges o_i . [1, 2, 5] Note that because all positively charged operators have vanishing VEVs, SUSY zero (holomorphic zero) mechanism act well, namely negatively charged interactions are forbidden by the anomalous $U(1)_A$ gauge symmetry. This SUSY zero mechanism plays an important role in solving the doublet-triplet splitting problem in a natural way, that is not discussed here.

3 Why M-theory?

Using anomalous $U(1)_A$ gauge symmetry, we can obtain GUT models in which doublet-triplet splitting is realized with generic interactions. However, it is inevitable in the scenario that mass spectrum of superheavy fields does not respect $SU(5)$ symmetry. Naively thinking, it spoils the success of gauge coupling unification in minimal $SU(5)$ GUT. In the early stage of our works, we expected that there must be a tuning parameter (charge) because the rank of $SO(10)$ or E_6 is larger than that of $SU(5)$. However, the fact was more exciting than what we expected.

Because in our scenario, mass spectrum of superheavy fields and VEVs (and unification scale $\Lambda_u \sim \lambda^{-a}$) are determined by $U(1)_A$ charges, gauge coupling unification conditions $\alpha_1(\Lambda_u) = \alpha_2(\Lambda_u) = \alpha_3(\Lambda_u)$ are rewritten by the $U(1)_A$ charges as

$$\Lambda \sim \Lambda_G, \quad (3.1)$$

$$h \sim 0. \quad (3.2)$$

It is amazing that all the charges except that of doublet Higgs are cancelled out in the conditions.[5] It means that there is no tuning parameters for the gauge coupling unification other than those in the minimal $SU(5)$ GUT. Actually, the first relation defines the scale of the theory and the second relation corresponds to that for the colored Higgs mass in the minimal $SU(5)$ GUT because the charge of doublet Higgs is the same as the charge of triplet Higgs.

Let us explain the situation. For any cutoff and for any charges, we can calculate the running gauge couplings $\tilde{g}_i(\Lambda_W)$ at the weak scale from the theory which is defined at the cutoff scale, in the GUT with anomalous $U(1)_A$. The above cancellation means that the three running gauge couplings $g_i(\mu)$, which are calculated in MSSM by using the gauge couplings $\tilde{g}_i(\Lambda_W)$ as the initial values at the weak scale, always meet at a scale, which is nothing but the cutoff scale in the scenario. The fact that three gauge couplings meet at the usual GUT scale $\Lambda_G \sim 2 \times 10^{16}$ GeV indicates that in the scenario of GUT with anomalous $U(1)_A$ the cutoff scale must be around the scale Λ_G . The real GUT scale is estimated as $\Lambda_U \sim \lambda^{-a} \Lambda_G$, which is smaller than the usual GUT scale. Therefore, proton decay via dimension 6 operators is one of the most interesting predictions in our scenario. If $a = -1$, the lifetime of proton is estimated as

$$\tau_p(p \rightarrow e\pi^0) \sim 1 \times 10^{34} \left(\frac{\Lambda_A}{5 \times 10^{15} \text{ GeV}} \right)^4 \left(\frac{0.01(\text{GeV})^3}{\alpha} \right)^2 \text{ yrs}, \quad (3.3)$$

using hadron matrix element parameter α calculated by lattice.[14] It is also quite interesting that these results are independent of the details of the Higgs sector. The sufficient conditions are just the following three conditions; 1, Unification group is simple. 2, The VEVs are given by eq. (2.5). 3, At a low energy scale, MSSM(+singlets) is realized.

What is important here for this talk is that the cutoff scale must be taken the usual GUT scale which is smaller than the string scale or Planck scale. Lowering the string scale is one of the most interesting subjects in the string phenomenology, and Horava-Witten pointed out that the extra-dimension in which only gravity modes can propagate realizes lower string scale in the context of heterotic M-theory.[10]

4 Why heterotic?

In our scenario, E_6 (or E_8) gauge group plays an important role in realizing bi-large neutrino mixings and in solving SUSY flavor problem by non-abelian horizontal symmetry. Therefore, if other type of superstring theory can induce E_6 (or E_8) gauge group, it is also considerable.

4.1 Bi-large neutrino mixings

First, it is explained that E_6 is important in realizing bi-large neutrino mixings. The fundamental representation of E_6 is divided as

$$\Psi_i(\mathbf{27}) \rightarrow \mathbf{16}_i[\mathbf{10}_i + \bar{\mathbf{5}}_i + \mathbf{1}_i] + \mathbf{10}_i[\bar{\mathbf{5}}_i + \bar{\mathbf{5}}'_i] + \mathbf{1}_i[\mathbf{1}_i] \quad (4.1)$$

under $(E_6 \supset) SO(10) [\supset SU(5)]$, where $i = 1, 2, 3$ is a generation index. What is important here is that there are six $\bar{\mathbf{5}}$ fields, three linear combination of which become massless. This structure gives different anomalous $U(1)_A$ charges of the massless $\bar{\mathbf{5}}$ fields from those of $\mathbf{10}$ fields, that results in different mixing matrices of quark and lepton. This is one of the most interesting feature in E_6 GUT, which Bando and Kugo called “E-twisting” structure.[15] Unfortunately, they examined special cases, in which SMA solution for solar neutrino problem is realized. However, as discussed in the previous section, in many models using $U(1)$ type Froggatt-Nielsen mechanism, LMA solution is the most natural solution for the solar neutrino problem. Moreover, this “E-twisting” structure naturally explain that the mixing angles of lepton sector become larger than those of quark sector. Roughly speaking, this is because $\bar{\mathbf{5}}_3$ and $\bar{\mathbf{5}}'_3$ naturally become superheavy because of the larger charge of third generation field Ψ_3 than those of the first 2 generation fields. Let us introduce the following interactions

$$\lambda^{\psi_i + \psi_j + \phi} \Psi_i \Psi_j \Phi + \lambda^{\psi_i + \psi_j + c} \Psi_i \Psi_j C, \quad (4.2)$$

where a non-vanishing VEV $\langle \Phi \rangle$ breaks E_6 into $SO(10)$ and gives mass terms of $\bar{\mathbf{5}}_i$ and $\bar{\mathbf{5}}_j$, and a non-vanishing VEV $\langle C \rangle$ breaks $SO(10)$ into $SU(5)$ and gives mass terms of $\bar{\mathbf{5}}_i$ and $\bar{\mathbf{5}}'_j$. It is natural to expect that $\bar{\mathbf{5}}_3$ and $\bar{\mathbf{5}}'_3$ become superheavy because $\psi_3 < \psi_2, \psi_1$. Therefore, main modes of three massless fields $\bar{\mathbf{5}}$ come from the first two generation fields, for example, $(\bar{\mathbf{5}}_1, \bar{\mathbf{5}}'_1, \bar{\mathbf{5}}_2)$. It is obvious that this results in milder hierarchy of $\bar{\mathbf{5}}$ charges than that of $\mathbf{10}$ charges. Therefore, larger mixing angles in lepton sector than those in quark sector are naturally obtained because the mixing angles are obtained as $V_{ij} \sim \lambda^{|c_i - c_j|}$, where c_i are q_i or l_i . And actually, realistic quark and lepton masses and mixings including bi-large neutrino mixings can be obtained in E_6 models as in the model in Table 1. In the model, the main component of doublet Higgs in MSSM comes from $\mathbf{10}_\Phi$ of $SO(10)$ and Yukawa couplings of $\bar{\mathbf{5}}'_1 + \lambda^\Delta \bar{\mathbf{5}}_3$ are mainly induced via the mixing $\lambda^\Delta \bar{\mathbf{5}}_3$, where $\Delta \sim 2.5$ is expected in the model in Table 1. Note that in E_6

GUT with anomalous $U(1)_A$, larger neutrino mixing angles than quark angles are automatically realized, while $SO(10)$ GUT scenario[1, 16] can reproduce bi-large neutrino mixings but not automatically.

4.2 Non-abelian horizontal symmetry as SUSY flavor problem

Second, it is shown that the fact that the massless $\bar{\mathbf{5}}$ fields come from the first two generation fields plays an important role in solving SUSY flavor problem by non-abelian horizontal symmetry.[7]

Before explaining the main point, we show that non-abelian horizontal gauge symmetry can be naturally embedded in the scenario of GUT with anomalous $U(1)_A$. [7] Pick up an example in Table 1. The $SU(2)_H$ breaking scale is determined by the VEV $\langle F^a \bar{F}_a \rangle \sim \lambda^{-(f+\bar{f})}$, which is given by the relation (2.5). The VEVs can be taken as $\langle F^T \rangle = \langle \bar{F} \rangle = (0, \lambda^{-\frac{1}{2}(f+\bar{f})})$, using the D -flatness condition of $SU(2)_H$ and $SU(2)_H$ gauge symmetry. Considering the $SU(2)_H \times U(1)_A$ invariants

$$\lambda^{\phi+f} \Psi^a \langle F^b \rangle \epsilon_{ab} \sim \lambda^{\phi-\frac{1}{2}(\bar{f}-f)} \Phi_1 \equiv \lambda^{\tilde{\phi}_1} \Phi_1, \quad (4.3)$$

$$\lambda^{\phi+\bar{f}} \Psi^a \langle \bar{F}_b \rangle \sim \lambda^{\phi+\frac{1}{2}(\bar{f}-f)} \Phi_2 \equiv \lambda^{\tilde{\phi}_2} \Phi_2, \quad (4.4)$$

$$\lambda^{\phi_3} \Phi_3 \equiv \lambda^{\tilde{\phi}_3} \Phi_3, \quad (4.5)$$

it is obvious that the effective charges $\tilde{\phi}_i$ ($i = 1, 2, 3$) determine the hierarchy in the superpotential. As a matter of fact, we can almost forget $SU(2)_H$ in considering the hierarchy of Yukawa couplings in the superpotential. Therefore, the discussion in the previous subsection is applicable even with non-abelian horizontal gauge symmetry. On the other hand, in the Kähler potential, $SU(2)_H$ symmetry leads to degenerate scalar fermion masses of the first two generation fields. The scalar fermion mass matrices of the model in Table 1 can be estimated as

$$\frac{\tilde{m}_{\mathbf{10}}^2}{\tilde{m}^2} \sim \begin{pmatrix} 1 + \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^5 & 1 + \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & O(1) \end{pmatrix}, \quad \frac{\tilde{m}_{\bar{\mathbf{5}}}^2}{\tilde{m}^2} \sim \begin{pmatrix} 1 + \lambda^4 & \lambda^6 & \lambda^5 \\ \lambda^6 & 1 + \lambda^2 & \lambda^7 \\ \lambda^5 & \lambda^7 & 1 + \lambda^4 \end{pmatrix} \quad (4.6)$$

where the corrections come from the interactions including F or \bar{F} , and the spurion field Z , which has non-vanishing F -term component $\langle F_Z \rangle = \tilde{m}\Lambda$, such as $K = |\Psi^a \bar{F}_a|^2 Z^\dagger Z$. It is quite impressive that all the three generation $\bar{\mathbf{5}}$ fields have degenerate scalar fermion masses in leading order because they come from the first two generation fields. This feature is important in suppressing FCNC. The various FCNC processes constrain the mixing matrices defined by $\delta_{\mathbf{R}} \equiv V_{\mathbf{R}}^\dagger \Delta_{\mathbf{R}} V_{\mathbf{R}}$ [17] ($\mathbf{R} = \mathbf{10}$ or $\bar{\mathbf{5}}$), where $\Delta_{\mathbf{R}} \equiv (\tilde{m}_{\mathbf{R}}^2/\tilde{m}^2) - 1$, $V_{\mathbf{10}} \sim V_{CKM}$ and $V_{\bar{\mathbf{5}}} \sim V_{MNS}$.

Here V_{CKM} is the Cabbibo-Kobayashi-Maskawa matrix and V_{MNS} is the Maki-Nakagawa-Sakata matrix. In this model, these mixing matrices are approximated as

$$\delta_{\mathbf{10}} = \begin{pmatrix} \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad \delta_{\bar{\mathbf{5}}} = \begin{pmatrix} \lambda^3 & \lambda^{2.5} & \lambda^3 \\ \lambda^{2.5} & \lambda^2 & \lambda^{2.5} \\ \lambda^3 & \lambda^{2.5} & \lambda^3 \end{pmatrix} \quad (4.7)$$

at the GUT scale. The constraints at the weak scale from ϵ_K in K meson mixing,

$$\sqrt{|\text{Im}(\delta_{d_L})_{12}(\delta_{d_R})_{12}|} \leq 2 \times 10^{-4} \left(\frac{\tilde{m}_q}{500 \text{ GeV}} \right) \quad (4.8)$$

$$|\text{Im}(\delta_{d_R})_{12}| \leq 1.5 \times 10^{-3} \left(\frac{\tilde{m}_q}{500 \text{ GeV}} \right), \quad (4.9)$$

requires scalar quark masses larger than 500 GeV, because in this model $\sqrt{|(\delta_{d_L})_{12}(\delta_{d_R})_{12}|} \sim \lambda^{7.5}(\eta_q)^{-1}$ and $|(\delta_{d_R})_{12}| \sim \lambda^{2.5}(\eta_q)^{-1}$, where we take a renormalization factor $\eta_q \sim 6$.¹ And the constraint from the $\mu \rightarrow e\gamma$ process,

$$|(\delta_{l_L})_{12}| \leq 4 \times 10^{-3} \left(\frac{\tilde{m}_l}{100 \text{ GeV}} \right)^2, \quad (4.10)$$

requires scalar lepton masses larger than 200 GeV, because $|(\delta_{l_L})_{12}| \sim \lambda^{2.5}$ in this model.

Note that in $SO(10)$ GUT with three $\mathbf{16}$ and one $\mathbf{10}_{[1]}$ in which one of the massless $\bar{\mathbf{5}}$ fields comes from the $\mathbf{10}$ of $SO(10)$, non-abelian horizontal symmetry does not guarantee the degenerate scalar fermion masses of three massless $\bar{\mathbf{5}}$ fields.

5 Discussion and summary

E_6 unified group is important in explaining larger mixings in lepton sector than in quark sector. Moreover, in E_6 GUT, introducing non-abelian horizontal gauge symmetry $SU(2)_H$ or $SU(3)_H$ can solve SUSY flavor problem even with bi-large neutrino mixings. In the resulting models, all the three generation quarks and leptons are unified into one or two multiplets, which is important in solving SUSY flavor problem.² Therefore, E_8 group looks promising because E_8 has a maximal subgroup $E_6 \times SU(3)_H$, though E_8 GUT cannot be realized in 4 dimensional theory because E_8 has no chiral representation.

¹The renormalization factor is strongly dependent on the ratio of the gaugino mass to the scalar fermion mass and the model below the GUT scale. If the model is MSSM and the ratio at the GUT scale is 1, then $\eta_q = 6 \sim 7$.

²Strictly speaking, the subgroup $SU(2)_E$ of E_6 , which rotates two $\bar{\mathbf{5}}$ fields in $\mathbf{27}$ of E_6 , is sufficient to realize this interesting structure. Therefore, the unified gauge group can be $SU(6) \times SU(2)_E$, $SU(3)^3$ or $SO(10)' \times U(1)$ [8]. The trification $SU(3)^3$ is interesting because doublet-triplet splitting is realized,[9] but this is not a simple group, so natural gauge coupling unification is not guaranteed.

In the scenario of GUT with anomalous $U(1)_A$ symmetry, the fact that in MSSM three gauge couplings meet at a scale Λ_G means that the cutoff scale of the theory must be the scale Λ_G . Because the scale Λ_G is smaller than the Planck scale, this may suggest that there are extra dimensions in which only gravity modes can propagate, as discussed by Horava-Witten.

Considering above two facts, GUT with anomalous $U(1)_A$ must imply heterotic M-theory.

Of course it is known that in weakly coupled heterotic string with Kaz-Moody level 1 cannot produce adjoint Higgs fields, which are required in our scenario. However, considering level 2 models, there are some models with $E_6 \times SU(2)_H$ with two E_6 adjoints.[18] Though these models do not include anomalous $U(1)_A$ gauge symmetry, the existence is quite impressive.

We hope that our study becomes a breakthrough in string phenomenology, and in near future, a path from the superstring to the standard model is discovered.

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